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INTERNAL SEICHES AND INTERFACIAL MIXING IN STRATIFIED LAKES

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by

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Abstract

The horizontal velocities of the upper and lower layers associated with an internal seiche episode in a stratified lake are examined in relation to critical shear gradients necessary for the growth of unstable short period interfacial waves with frequency close to the so-called Brunt-Vaisala frequency. Experimental results are presented which help to clarify the conditions for the occurrence of the short period waves. Charts summarizing the findings of this study are presented which should be of help in predicting the occurrence of internal wave breaking in closed basins. It is believed that interfacial wave breaking in a stratified lake increases vertical mixing between the epilimnion and hypoliminon and therefore may be a factor in the seasonal development of the thermocline and in the distribution of dissolved and suspended substances.

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I. Introduction

It is well known that a stable density stratification in a fluid environment inhibits vertical mixing and can act as an effective barrier to the transport of mass, momentum, and energy across the interface where the density variation occurs. Oceans, lakes, and reservoirs become stably stratified due to naturally occurring seasonal variations in their heat budget. Other causes of density variations can be attributed to the presence of dissolved or suspended substances and to the introduction of waste heat. These latter causes can produce significant local alterations in the currents and mixing processes of the natural environment but the seasonal stratification causes profound changes in the large scale circulation patterns and in the vertical distribution of suspended and dissolved substances. For practical purposes, seasonal stratification produces a two-layered system in which vertical exchange between layers has been significantly reduced. This condition has important implications with regard to the distribution of dissolved and suspended substances; in particular, the distribution of dissolved oxygen.

One important hydrodynamic phenomenon occurring at the interfacial region is internal waves. The interfacial shear caused by internal waves tends to increase vertical turbulent diffusion over that due to just diffusive mechanisms in the absence of internal waves. Internal wave breaking would further increase the vertical mixing occurring at the interface.

This report deals mainly with closed basins such as lakes and reservoirs and with the occurrence of the internal long wave resonant

condition commonly referred to as the internal seiche. The horizontal velocities associated with an internal seiche episode are examined in relation to critical shear gradients necessary for the growth of unstable short period waves with frequency close to the so-called Brunt-Vaisala frequency. Some experimental results are presented which help to clarify the conditions for the occurrence of the short period waves. Charts summarizing the findings of this study are presented which should be of help in predicting the occurrence of the short period unstable wave growth in closed basins.

The local shear at the interface caused by the oppositely directed translational currents in the upper and lower layers during a seiche episode can also be caused by the flow of one layer alone. Examples of this would be the flow of a silt laden "density" current into a reservoir or the flow of a warmer layer of water over a more dense colder bottom layer. The latter has received increased attention as a result of the anticipated rise of thermal loading of natural bodies of water from new electric-generating power plant development.

Direct observation in the field of the breaking of short-period internal waves has been lacking. A temperature reversal just above the main thermocline in a lake could be attributed to a breaking wave. Such bathythermograph profiles have been recorded [Stewart (1971)]. Fig. 1 shows schematically how this could occur.

Woods (1968) was able to photograph underwater the breaking of short-period waves which were superimposed on longer internal waves in the summer thermocline of the Mediterranean Sea. These breaking waves were shown to be of the shear-generated instability type and were found to occur in thin laminar layers or sheets having sharp

temperature gradients. These sheets occurred at spaced depth intervals within the main summer thermocline and were reported to be a typical phenomenon of the Mediterranean Sea.

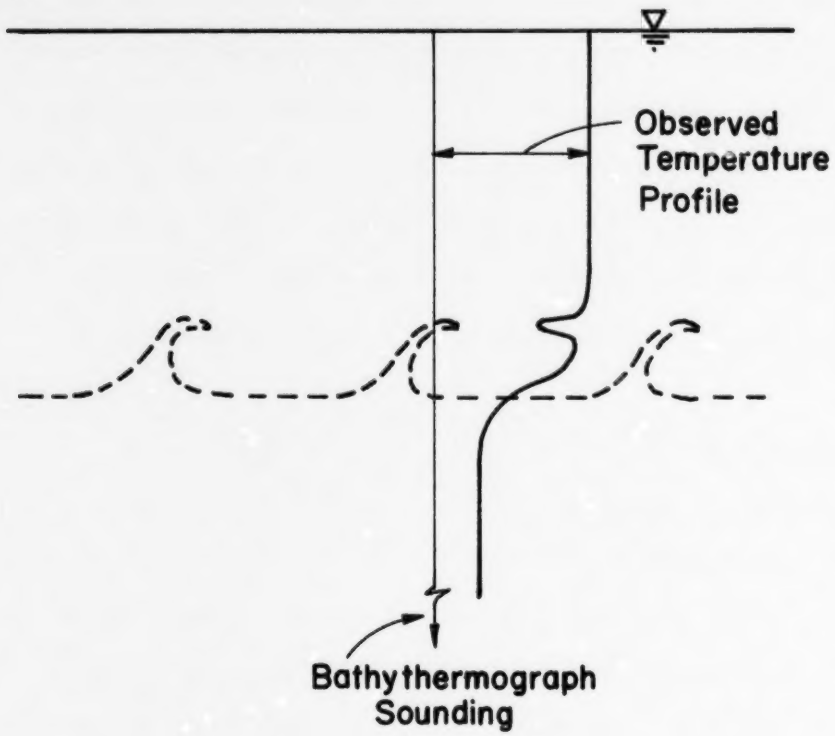

Fig. 1. Temperature Reversal Caused by Breaking Internal Wave

There is no doubt that the shear-generated breaking wave phenomenon occurs in lakes, oceans, and reservoirs. The frequency of its occurrence is not known and the role it plays in vertical mixing and in the control of the development of the thermocline is worthy of study. A similar phenomenon in the atmosphere having the same physical origin is one of the reasons for the so-called "clear air turbulence," [Townsend (1968)].

Appendix A to this report discusses the requirements for dynamic similarity between model and prototype in the study of interfacial hydrodynamics. An examination of the criteria for similitude points up the difficulties in attempting model studies in which it is hoped to simulate the interfacial hydrodynamics of a lake or reservoir.

Some preliminary work on interfacial friction is reported in Appendix B.

II. Review of Theory

A. Interfacial Instability

1. Inviscid Flow

Shear-generated short-period internal gravity waves caused by the oppositely directed translatory currents of a stably stratified two-layered closed basin are hypothesized to be of the Brunt-Vaisala type. It has been shown [Groen (1948), Heyna and Groen (1958)] that the shortest period internal wave, T_N, that can occur in a stratified incompressible fluid is given by

$$T_N = \frac{2\pi}{N} \tag{1}$$

where

$$N = \sqrt{\frac{g}{\rho}\frac{\partial \rho}{\partial z}}, \tag{2}$$

in which

N = Brunt-Vaisala frequency

g = gravitational acceleration

ρ = local fluid density at position z

$\frac{\partial \rho}{\partial z}$ = rate of change of density with elevation at position z.

Apparently Brunt and Vaisala independently determined that the period of oscillation of a fluid particle displaced from its position of neutral buoyancy would be given by Eq.(1) [Mortimer (1968)]. Fig. 2 portrays schematically such a situation. An elementary force balance analysis neglecting friction and assuming a periodic-type solution will lead to the specification of N as the natural frequency of oscillation of the displaced particle.

An excellent review of the work of Taylor (1931), Goldstein (1931) and others on the stability of stratified flows is given by Yih (1965). For two superposed streams of different density separated by a transition region of thickness, δ, in which the velocity varies linearly and the density is the mean of the bottom and top layers, Goldstein determines a shaded region of instability as shown in Fig. 3. In Fig. 3, α is the dimensionless wave number where λ is the wave length, and R_i is the Richardson number, $R_i = g'\delta/U^2$, where $g' = g\,\Delta\rho/\bar{\rho}$ and U is the velocity difference across the interface region. Miles and Howard (1964) using a model similar to Goldstein's but with a linear variation of density in the intermediate layer, found a much smaller domain of instability in the R_i, α plane.

One can conclude from the work on hydrodynamic stability of stratified flows that when the local value of the Richardson number is less than some critical value, the flow will be unstable to disturbances

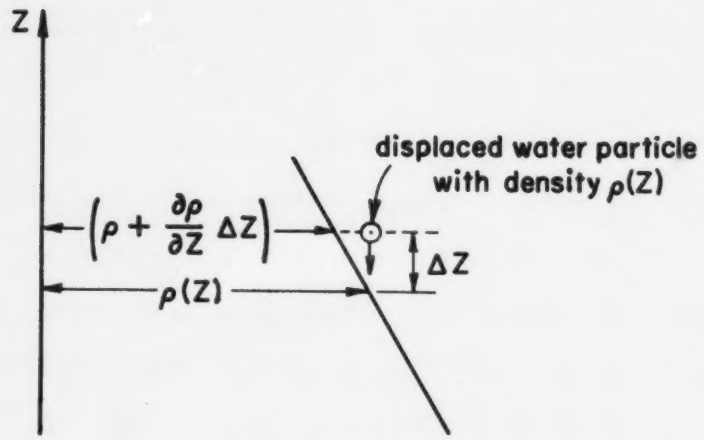

Fig. 2. Schematic Sketch of Water Particle Displaced from Neutral Position in a Variable Density Field

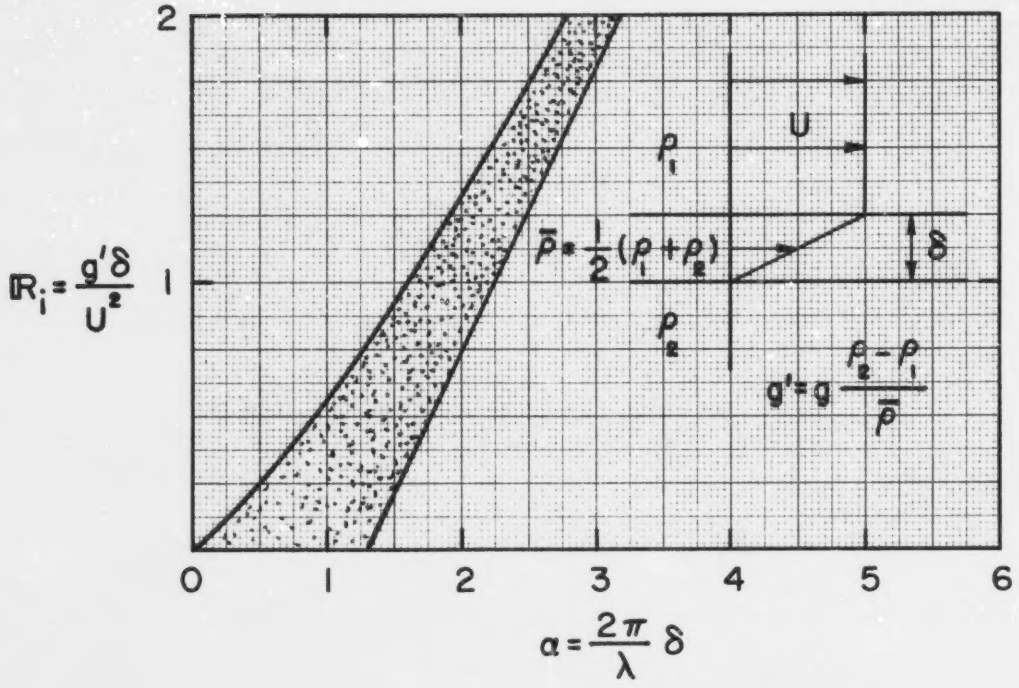

Fig. 3. Interfacial Instability Chart (after Goldstein, 1931)

that fall within a certain wave length range. The actual critical value of the Richardson number and the range of wave lengths are probably dependent on the details of the velocity field including the turbulence characteristics.

The celerity of a short period internal wave relative to the mean motion (for small density difference) is given by [Keulegan (1950)]:

$$C = \sqrt{\frac{g'\lambda}{4\pi}} \tag{3}$$

If it is assumed that the celerity of the unstable waves forming in the interfacial region move with one half the velocity of the upper and lower streams (i.e., $C = U/2$ where $U = u_1 - u_2$, see Fig. 4), and if it is further assumed that the period of these waves is given by Eq.(1), then the following expression relating the wavelength, λ, and the interface thickness, δ, is obtained:

$$\lambda = \pi\delta \tag{4}$$

and it then follows that the critical shearing velocity is

$$U_c = \sqrt{g'\delta} \tag{5}$$

Eqs. (4) and (5) give a critical Richardson number equal to one ($R_i = 1$) and wave number equal to 2 ($\alpha = 2$). Justification for the celerity of the shear-generated waves of instability comes from the experimental and analytical work of Keulegan (1949).

Based on this analysis, one would expect that for $R_i \leq 1$, the flow in the region of the interface would be conducive for waves to be unstable, grow in amplitude and break. This approximate analysis is an attempt to provide a rational basis for the prediction of interfacial wave breaking in the field. The foregoing discussion assumes no internal resistance in the determination of critical conditions for incipient wave breaking.

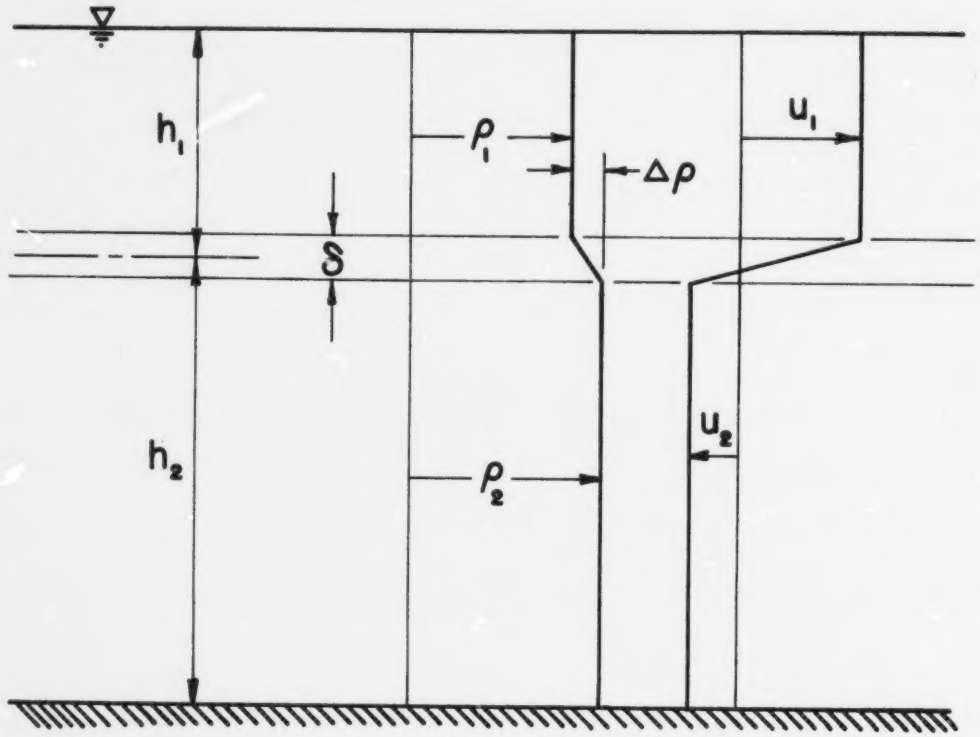

Fig. 4 Schematic Sketch of Idealized Two-Layered System

2. Viscous Flow

Keulegan (1949) introduced a parameter, θ, given by

$$\theta = \frac{(\nu_2 g')^{1/3}}{U} \tag{6}$$

which he derived based on an analysis of the equations of motion including viscosity. The analysis was similar to that given by Jeffreys (1926) in his study of the critical wind speed necessary for the generation of short-period gravity water waves. It can be seen that the parameter, θ, is related to the Richardson number and an interfacial Reynolds number as follows:

$$\theta = \left[\frac{R_i}{R_\delta} \right]^{1/3} \tag{7}$$

where $R_\delta = U\delta/\nu_2$.

It then follows that for a given critical Richardson number the parameter, θ, is given by

$$\theta_c = (\text{constant})\ R_\delta^{-1/3} \tag{8}$$

For $R_i = 1$, the constant in Eq.(8) is one and for $R_i = 1/4$, the constant is 0.63.

Ippen and Harleman (1952) concluded that a relationship similar to Eq.(8) (in which δ was replaced by h_2 in the Reynolds number), was indeed valid in the laminar flow regime. Their analysis was based on an assumption that a Froude number given by $F = U/\sqrt{g'h_2}$ would be equal to one at the critical condition when interfacial surface disturbances become unstable. Generally, one would not expect the depths of the two layers to be of great importance in the determination of interfacial instability unless perhaps, one of the layers was very shallow. In the experiments reported by Ippen and Harleman, the depth h_2 was very small and would have been on the same order as δ.

In general, it would seem that only the Richardson number and the Reynolds number would be important in specifying the conditions for interfacial instability. These two parameters can be grouped to form a third parameter, θ, if δ is eliminated as a variable. As mentioned previously, the scale and intensity of turbulence in either layer could have an appreciable effect on the stability of the interfacial region. One could imagine that if either of the flows in the upper or lower layers contained a high amount of turbulent energy at a frequency very close to the Brunt-Vaisala frequency, then the tendency for interfacial instability would be greater and could occur at higher Richardson numbers.

For this reason it is anticipated that the parameter, θ, may exhibit some dependence on the Reynolds numbers of the outer flows, i.e. $R_1 = U_1 h_1/\nu_1$ and $R_2 = U_2 h_2/\nu_2$.

It is clear that in the field distinct two-layered situations will not always exist and the exact specification of δ and therefore g' may be uncertain. The overall dimensions of the thermocline region may not be appropriate as Woods (1968) has pointed out. The finer structure of the thermocline region may give rise to local instability long before conditions are such that the entire thermocline region becomes unstable. In those cases, a detailed examination of the density profile would be necessary for prediction of interfacial instability and internal wave breaking.

B. Internal Seiches

The idealized physical situation under consideration is shown in Fig. 5. Stokes (1847) dealt with this case assuming inviscid flow and an abrupt change in density at the interface between the upper and lower layers. Utilizing a velocity potential for the upper layer and one for

the lower layer, he was able to derive a general expression for interfacial wave celerity providing the amplitude of the wave was small. When the difference in density between the layers is small and when the wave length of the internal wave is long compared with the depth of either layer, the celerity, C, of the internal standing long wave can be closely approximated by

$$C = \sqrt{g' \frac{h_1 h_2}{h_1 + h_2}} \tag{9}$$

Furthermore, the maximum horizontal velocity, u_1, of the upper layer is given by

$$u_1 = \frac{a_2}{h_1} C \tag{10}$$

and for the lower layer

$$u_2 = \frac{a_2}{h_2} C \tag{11}$$

where a_2 is the maximum amplitude of the internal wave. The ratio of the amplitudes of the standing internal wave, a_2, and the corresponding standing surface wave, a_1, is given by

$$\frac{a_2}{a_1} = \frac{\rho_2}{\Delta\rho} \cdot \frac{h}{h_2} \tag{12}$$

where $h = h_1 + h_2$.

From Eq.(12) it can be seen that under ordinary circumstances, a_2 will be very much greater than a_1. Hunt (1961) and Thorpe (1968) have considered higher order theories for the finite amplitude internal wave.

For a closed basin of length, ℓ, the period of the uninodal internal seiche will be

$$T_s = \frac{2\ell}{C} \tag{13}$$

and the maximum velocities given by Eqs.(10) and (11) will occur at the node position.

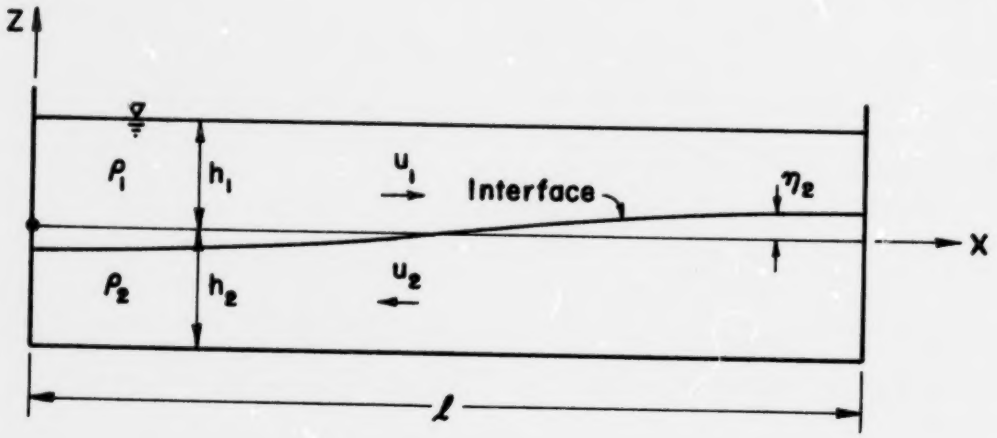

Fig. 5 Schematic Sketch of Uninodal Internal Seiche

It has been shown [Mortimer (1951)] that the two-layered model considered above predicts periods for the internal seiche in close agreement with field observations, even in the presence of an interfacial transition region of thickness, δ, which must always exist. The interfacial region thus becomes the shearing layer between the oppositely directed oscillatory motion of the upper and lower layers.

The region of highest interfacial shear will occur at the nodal position. From inspection of Eqs. (10) and (11), the shearing velocity, U, can be written as

$$U = a_2 \sqrt{g' \frac{h_1 + h_2}{h_1 h_2}} \qquad (14)$$

Assuming an interfacial thickness, δ, small in comparison to h_1 and h_2, the interfacial Richardson number will be a minimum at the nodal position and will be given by

$$R_{i_{min}} = \frac{\delta h_1 / a_2^2}{\frac{h_1}{h_2} + 1} \qquad (15)$$

From Eq.(15), we see that the two most important factors contributing to small Richardson numbers will be large internal wave amplitudes and small interfacial thicknesses. Eq.(15) is based on small amplitude wave theory and will only give approximations for large amplitude internal waves. Observations in Cayuga Lake, N.Y. [Henson et al. (1961)] show that h_1/a_2 can approach one under extreme conditions and δ/a_2 can become as small as 1/10 or 1/20. Under such conditions the Richardson number would be much less than one as presented here and conditions would be favorable for interfacial wave breaking and rapid mixing in the nodal region.

III. Experimental Program

A. Experimental Set-up

At first it was hoped that internal seiches generated in a laboratory flume closed at both ends would produce a sufficiently high shear velocity, U, that interfacial waves and the resulting interfacial mixing could be observed directly but at a reduced scale. A flume with a closed basin 358 cm in length, 15.3 cm in width, and 45 cm deep was supported in such a way that it could be rocked about a pivot in almost sinusoidal motion. The amplitude and period of this forced motion could be varied. Two layered systems of fresh and salt water were rocked at frequencies close to the natural period of the uninodal internal seiche [Eq.(13)]. Although it was possible to tune the flume oscillation to the sloshing (or first) mode of the lower layer, secondary progressive gravity waves formed at the interface and were superimposed on the standing long wave. In order to avoid the secondary wave forms, internal seiche amplitudes had to be kept small and this reduced the interfacial shear. It was not possible to obtain instability at the nodal position under this condition and measurements of salt concentration profiles led to the conclusion that interfacial mixing was governed mainly by molecular diffusion.

The experimental apparatus was then modified to produce a steady non-uniform flow of the upper layer over a quiescent heavier bottom layer (see Fig. 6). A series of experiments was conducted to measure the critical velocity, U_c, of the upper layer that brought about the condition of incipient wave breaking at the interface.

Measurements of salt concentration profiles in the region of the interface were made with a conductivity probe (electrode dimensions were

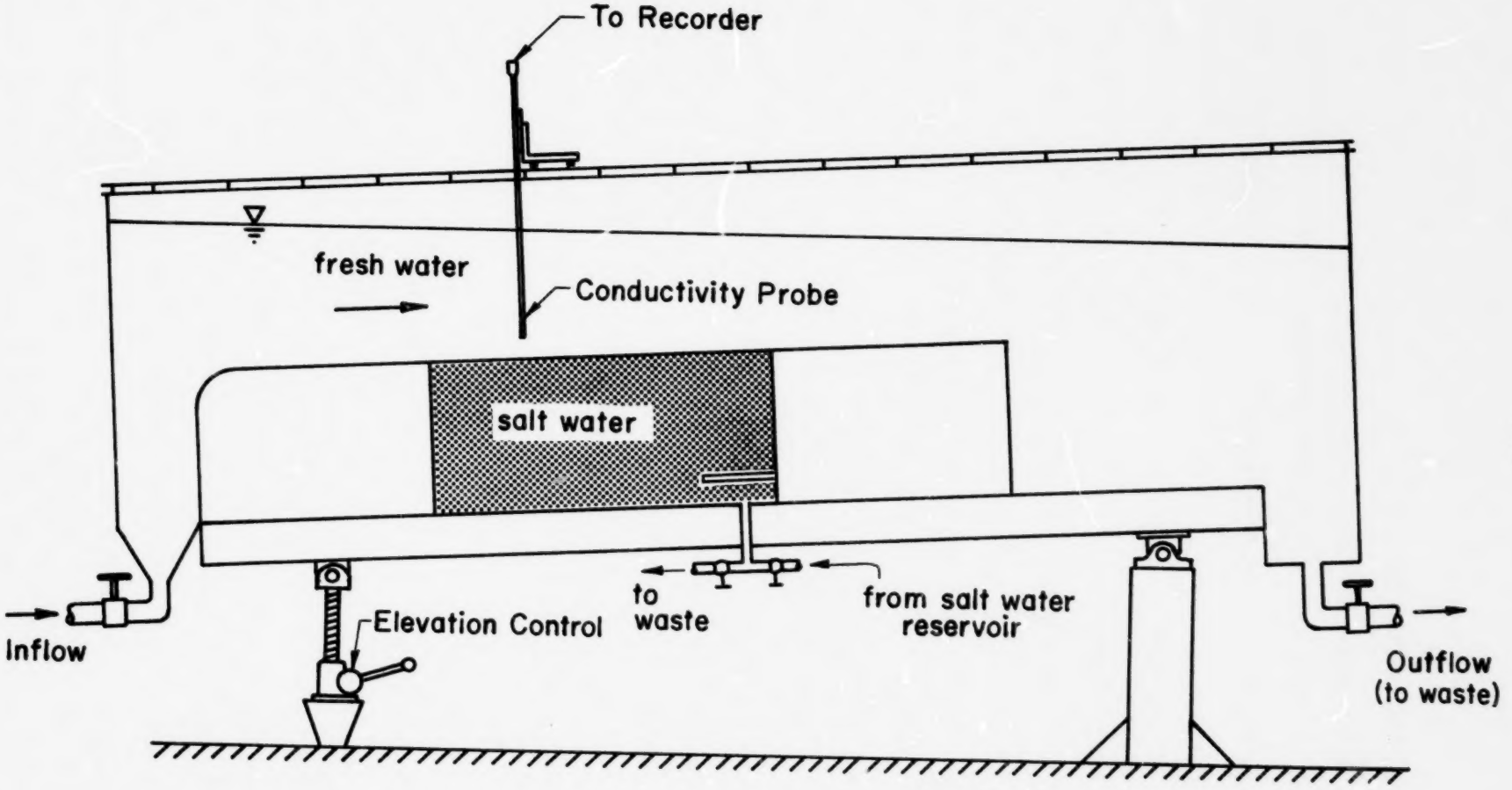

Fig. 6 Schematic Diagram of Experimental Set-Up for Study of Interfacial Instability

0.32 cm x 0.32 cm) and recorded on a Sanborn oscillograph [see Ditmars (1970)]. The measurements were made just prior to the formation and breaking of wavelets. The experimental data are recorded in Table I. In subsequent experiments it would be desirable to utilize a probe with much smaller electrodes since the interfacial thicknesses were on the order of twice the dimensions of the electrodes used.

The volume rate of flow of the upper layer was measured gravimetrically and U_c was obtained by dividing this discharge by the appropriate cross sectional area. Occasionally, checks on this velocity were made by timing the rate of motion of particles suspended in the flow. The temperature of the upper layer flow was adjusted to be the same as the static lower layer.

Observations of the interface were made visually. The lower layer contained a dark blue dye and there was no difficulty in observing the formation of wavelets and wave breaking. The determination of the velocity that produced incipient wave breaking at the interface was somewhat subjective. For this series of experiments, incipient wave breaking was defined as the condition when there was intermittent formation of breaking wavelet groups (see Fig. 7). The observed conditions at the interface were as described by Keulegan (1949) and his results are comparable in every way to those reported on here. The main difference is that Keulegan did not make measurements of the thickness of the interface region.

The depth of the lower layer was fixed at 20 cm. The density of the lower layer was adjusted by changing the concentration of dissolved sodium chloride.

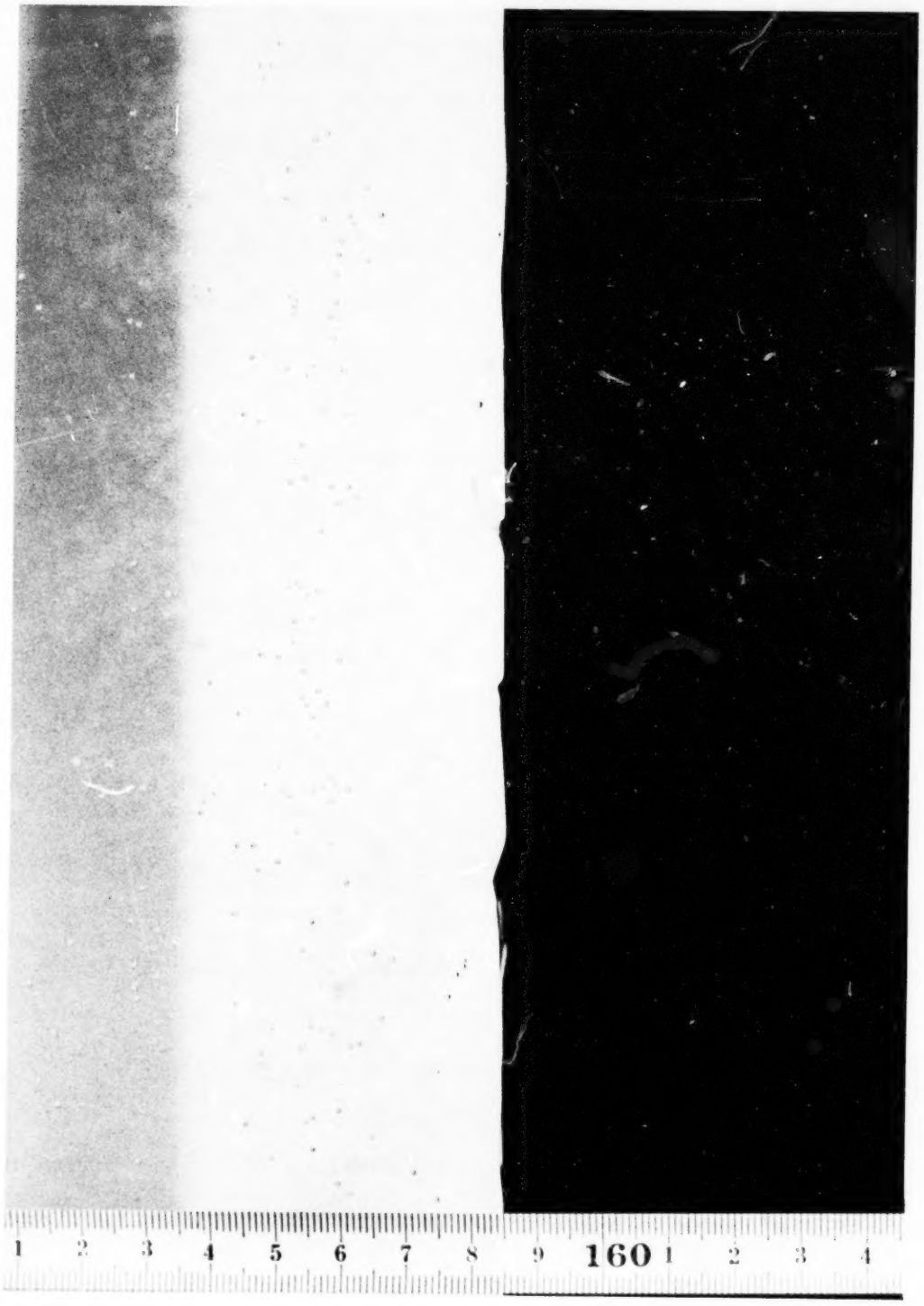

Fig. 7 Photograph of Breaking Wavelet Group

B. Presentation of Results

The experimental results are summarized in Table I. The Reynolds number R_e has been calculated according to $R_e = 4h_r U_c/\nu_1$, where the hydraulic radius, $h_r = h_1 b/(2h_1 + b)$ and b is the flume width. The data of Keulegan (1949) and Ippen and Harleman (1952) are included in the plot of the stability parameter, θ, as a function of the flow Reynolds number, R_e (see Fig. 8). The data listed in Table I corroborate the early work of Keulegan (1949) over the limited range of Reynolds numbers.

Table I

Test Results for Incipient Wave Breaking Experiments

Test	h_1 (cm)	U_c (cm/sec)	δ (cm)	ρ_1 (gm/cm^3)	ρ_2 (gm/cm^3)	ν_2 (cm^2/sec)	R_δ	R_i	R_e	θ	F*
I-A	11.0	5.80	.45	1.000	1.075	.0114	228	.944	10,450	.163	.208
I-B	10.3	5.53	.40	1.000	1.065	.0112	198	.810	9,700	.162	.220
I-C	16.5	4.13	.35	1.000	1.046	.0110	132	.905	8,640	.192	.154
I-D	17.0	3.88	.45	1.001	1.034	.0102	172	.950	8,200	.178	.168
I-E	17.5	3.72	.40	1.000	1.025	.0100	150	.686	7,920	.167	.182
I-F	16.1	3.26	.42	1.000	1.016	.0099	138	.618	6,770	.165	.205
I-G	13.5	3.03	.47	1.001	1.010	.0098	146	.450	5,830	.146	.278
I-H	13.7	2.00	.50	1.001	1.005	.0098	103	.490	3,930	.169	.274

*$F = U_c/\sqrt{g'h_1}$

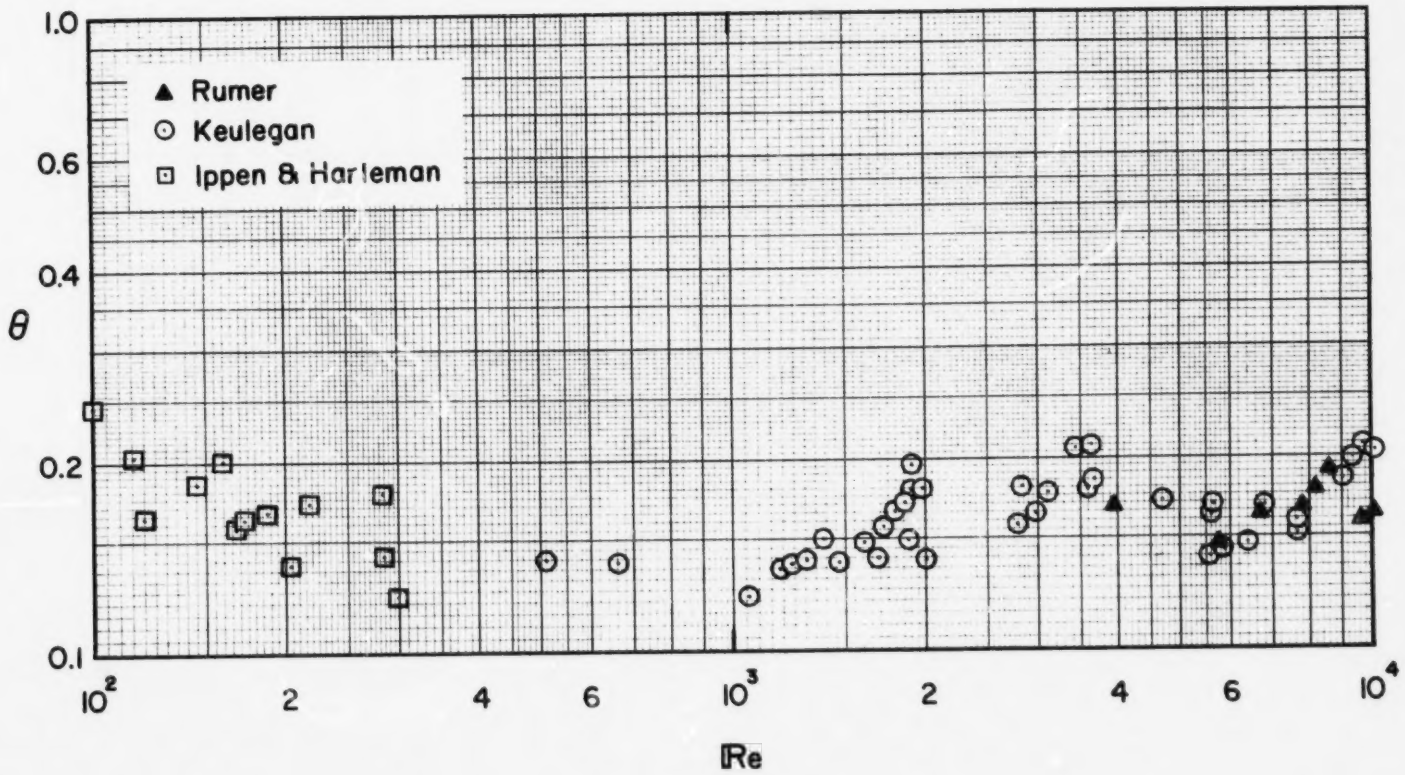

Fig. 8 Stability Chart for Incipient Interfacial Wave Breaking

The data are also plotted in Fig. 9 where it is seen that Eq.(5) gives a lower bound on critical velocities necessary for incipient wave breaking.

The data support the hypothesis that the stability of the interface is dependent on the initial unstable wave forms having a frequency close to the Brunt-Vaisala frequency. The stability of the interface seems to be independent of the depths of the upper or lower layers. The densimetric Froude number, F, of the upper layer is much less than 1, averaging 0.211. There is no reason why the upper layer Froude number, F, should take on a constant value at critical conditions when wave breaking first occurs. In fact, if the Richardson number is constant at critical conditions, then since $F = \sqrt{\frac{\delta/h_1}{R_i}}$, the Froude number would vary with the ratio of the interface thickness and the upper layer thickness.

Thorpe (1971) has presented some experimental results in which the wavelength, λ, of the interfacial wavelets was correlated with the interfacial transition thickness, δ. His data are replotted in Fig. 10 with Eq.(4) plotted for reference purposes. Eq.(4) (in which $\lambda = \pi\delta$) is by no means a perfect fit to the data of Thorpe but it does seem to provide a lower bound and therefore supports the hypothesis that interfacial instability is intimately related to the natural frequency of oscillation of displaced fluid particles in a nonhomogeneous density field.

As pointed out earlier, the stability paramter, θ, can be written as a function of the Richardson number and a Reynolds number, R_δ, where the interface thickness, δ, is used as the characteristic length. If it were assumed that at critical conditions the Richardson number takes

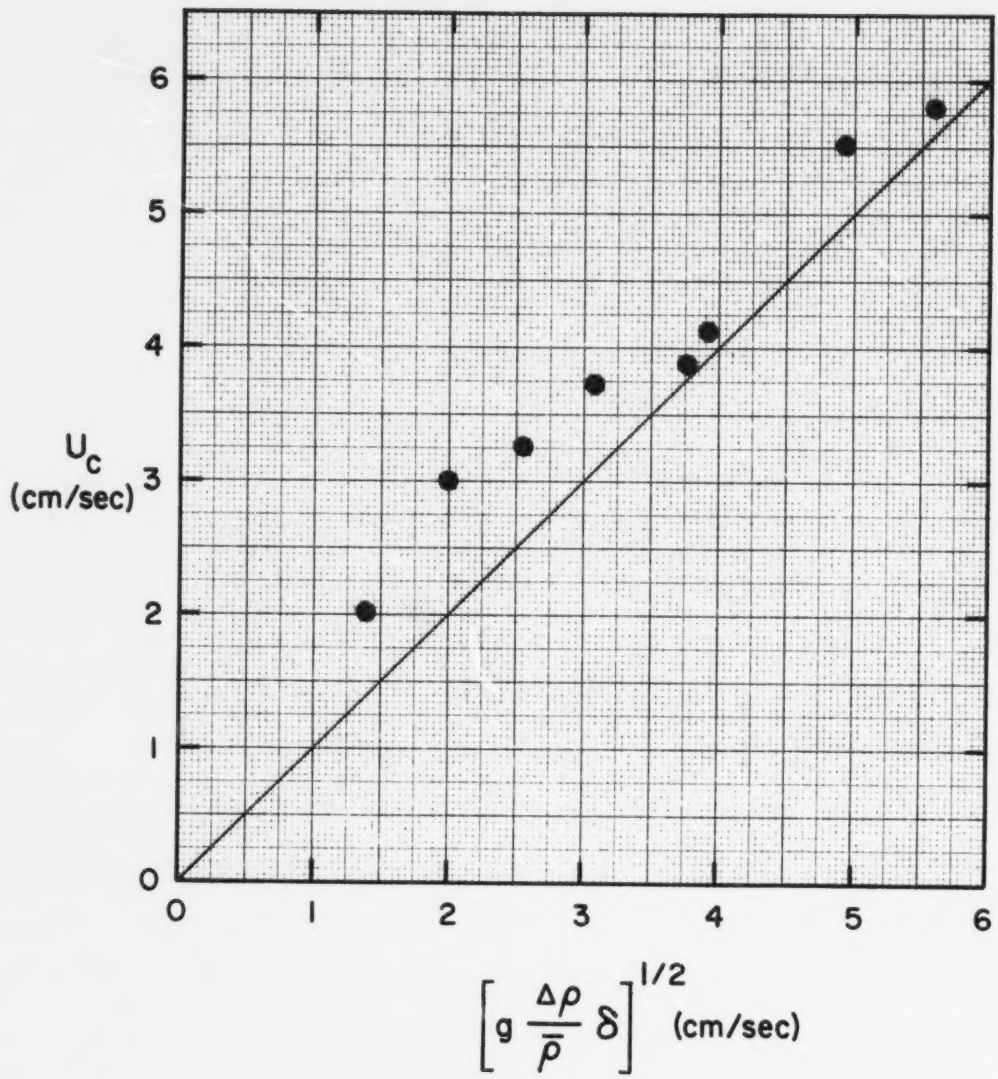

Fig. 9 Experimental Results: Critical Shearing Velocity for Instability

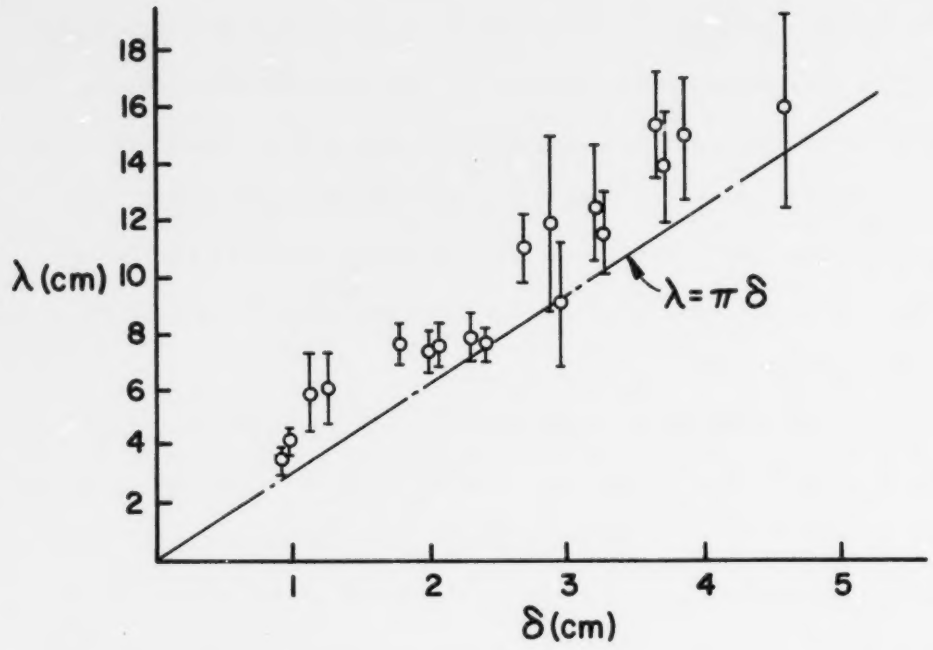

Fig. 10 The Variation of Wavelength, λ, with Interfacial Thickness, δ [after Thorpe (1971)]

on a constant value (or nearly constant value, then it is seen from Eq.(8) that θ will still vary with R_δ. This conclusion does not seem to be supported by the available data and at least over the range of Reynolds numbers investigated, θ appears to vary only slightly and could almost be considered as constant. For instance, in Fig. 8, over a Reynolds number range covering two orders of magnitude, θ varies from 0.12 to 0.21 with only one experimental data point falling outside of this range. The influence of turbulence in the upper and lower layer flows (and therefore the flow Reynolds number, $R_e = Uh_1/\nu_1$) on the onset of interfacial instability needs further study.

IV. Discussion

Based on the findings of this study it is concluded that wherever the incerfacial layer Richardson number is less than or equal to one, instability is imminent and breaking wavelets may occur. This criterion provides a rational basis for predicting the circumstances under which interfacial mixing due to shear-generated breaking wavelets may occur in the field or laboratory. Fig. 11 has been presented as an aid to making such a predicition.

If the density profile is known (based on temperature records) estimates of $\Delta\sigma$ and δ can be made ($\Delta\sigma = 1000\ \Delta\rho/\rho$) and the coordinate point ($\Delta\sigma$, δ) can be located on the figure. The slanted parallel lines representing constant velocity are representations of Eq. (5). If the shearing velocity across the layer is known to exceed the velocity indicated by Fig. 11, then the Richardson number is less than one and interfacial mixing due to internal wave breaking may be expected. In order to predict the possible shearing velocity to be expected from an internal seiche, Eq. (14) can be used. Some estimate or knowledge of the amplitude of the internal seiche is necessary.

As an example an internal seiche episode recorded in Cayuga Lake, N.Y. [Henson et al (1961)] provides the following data: δ = 16 m, $\Delta\sigma$ = 1.8, h_1 = 16 m, h_2 = 41 m and a_2 = 15 m. From Fig. 11 it is seen that the critical shearing velocity would be somewhat greater than 50 cm/sec. The shearing velocity calculated from Eq. (14) gives U = 59 cm/sec for the recorded seiche. Although only approximate, this analysis would indicate that internal wave breaking may have occurred in the nodal region. The amplitudes of the breaking waves would have been relatively large. The interfacial layer thickness is quite large

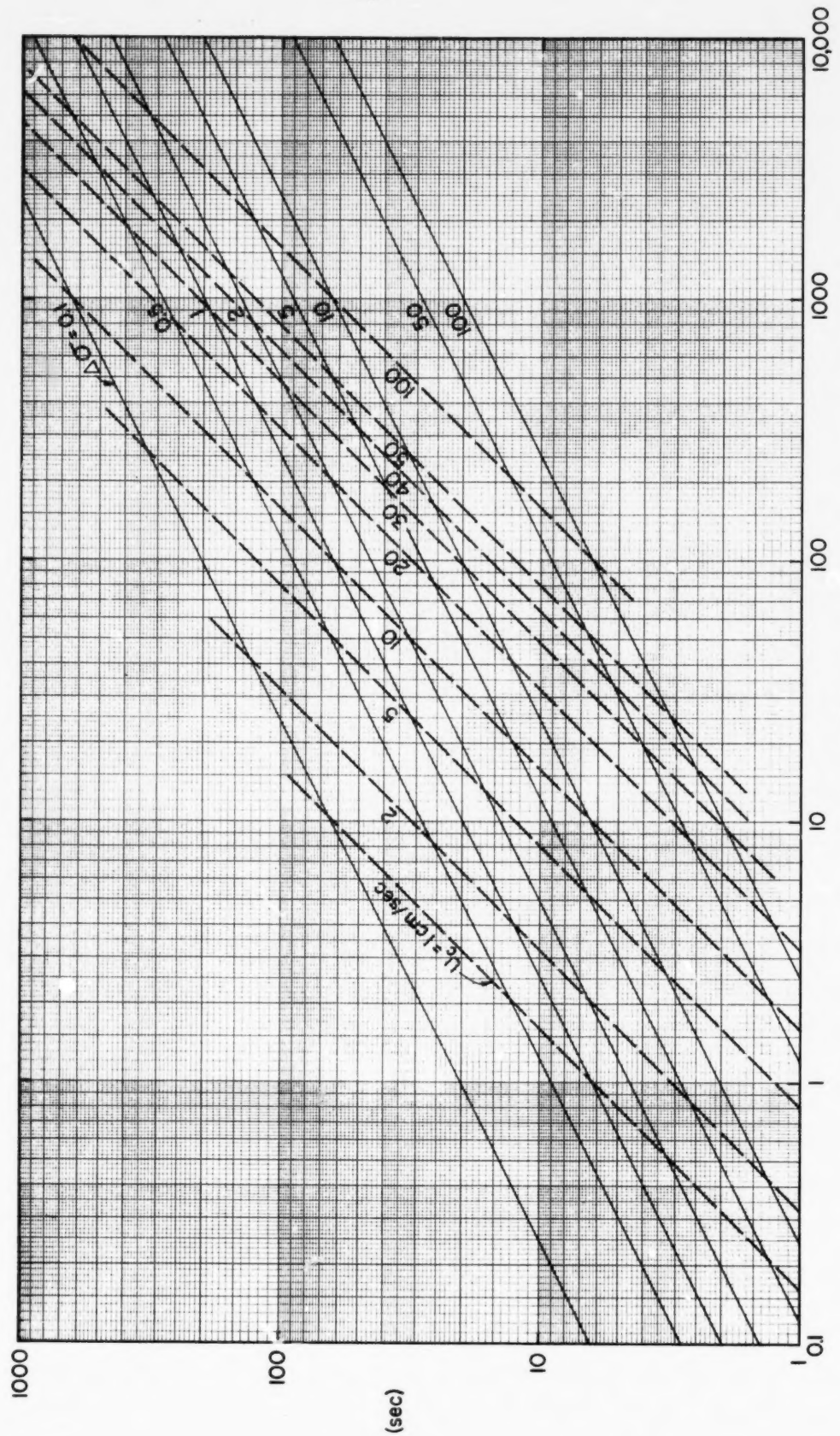

Fig. 11 Interfacial Instability Chart: Critical Shearing Velocity as a Function of Interfacial Thickness and Density Difference (based on $R_i = 1$, eqns. (3), (4), and (5) and g=980 cm/sec^2)

for this example and it is likely that the isotherms may have converged in certain parts of the lake, thus giving a smaller δ and a greater likelihood of interfacial instability. Other data [Henson et al (1961)] show that for Cayuga Lake, δ can vary from 2 m to 20 m during the stratification period while $\Delta\sigma$ varies from 0.5 to 1.8. These ranges are for the condition when the lake can be represented as a well-defined two layered system.

The laboratory attempt at generating interfacial instability by modelling the internal seiche gave the following data: δ = 1.3 cm, $\Delta\sigma$ = 63, h_1 = 3 cm, h_2 = 37.65 cm, a_2 = 1 cm. From Fig. 11 the critical velocity necessary would have been approximately 9 cm/sec. The calculated velocity from Eq.(14) is 4.7 cm/sec which is less than the critical. Actually, the horizontal velocity was less than that given by Eq.(14) because the motion was forced at a period less than the natural period for the uninodal internal seiche as explained earlier.

All of the above is of interest mainly because of the expected increase in interfacial mixing to be expected when internal wave breaking occurs. Preliminary attempts were made at measuring density profiles (or salt concentrations) in the interfacial region when regular wave breaking was occurring. The experimental set-up was as shown in Fig. 6. The recorded conductivity at a fixed position had a temporal variation not unlike the fluctuation of a turbulent velocity component. Careful experiments in which the conductivity measurements were regularly sampled over short time periods, digitized and recorded on magnetic tape would provide data for a meaningful analysis of interfacial mixing. As the crest of a breaking wave penetrates into the flowing upper layer, it is partially mixed by the turbulent eddies in that layer. Any theoretical

analysis of this problem should include this aspect.

As more knowledge is gained concerning the mixing to be expected from breaking interfacial waves and as the circumstances for occurrence of this phenomenon are better understood, it will be possible to improve our understanding of the transfer of mass, momentum and energy between the epilimnion and the hypolimnion of a stratified lake. Such understanding may, for example, lead to more effective management of liquid thermal wastes that are to be discharged into lakes. Theoretical models for the oxygen balance in a stratified lake will also be improved as we are able to better predict the oxygen transfer across the interfacial region.

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Notation

a_1	upper layer amplitude
a_2	amplitude of internal seiche
b	flume width
C	celerity of internal seiche
F	Froude number ($= U/\sqrt{g'h_1}$
g	gravitational acceleration
g'	$g\,\Delta\rho/\bar{\rho}$
h_1	depth of upper layer
h_2	depth of lower layer
h	$h_1 + h_2$
h_r	hydraulic radius ($= h_1 b/(2h_1 + b)$)
ℓ	basin length
N	Brunt-Vaisala frequency
R_e	Reynolds number ($= U4h_r/\nu_1$)
R_i	Richardson number ($= g'\delta/U^2$)
R_δ	Reynolds number ($= U\delta/\nu_2$)
T_N	Brunt-Vaisala wave period
T_S	internal seiche period for closed basins
u_1	maximum horizontal seiche velocity in upper layer
u_2	maximum horizontal seiche velocity in lower layer
U	$u_1 - u_2$
U_c	critical value of U causing interfacial wave breaking
Z	vertical coordinate direction
α	dimensionless wave number ($= 2\pi\delta/\lambda$)
δ	thickness of interfacial transition region

θ	stability parameter ($= \nu_2 g')^{1/3}/U$)
λ	wave length
ν	kinematic viscosity
ρ	density of liquid
$\Delta\sigma$	density difference parameter ($= 1000\,\Delta\rho/\bar{\rho}$
$\Delta\rho$	difference in density between upper and lower layers of stratified lake ($= \rho_2 - \rho_1$)

APPENDIX A

Criteria for Modelling

In this section the hydraulic modelling of the interfacial hydrodynamics of a prototype lake will be considered. It will be assumed that the entire lake system is also being simulated in the same physical model and therefore, that modelling criteria for the former must be compatible with the criteria for the latter.

The similtude criteria for a vertically distorted model of a stratified prototype lake is given as [Rumer and Hoopes (1970)]:

$$U_r = [Z_r]^{\frac{1}{2}} \tag{A-1}$$

$$U_r = \left[\left(\frac{\Delta\rho}{\rho}\right)_r Z_r\right]^{\frac{1}{2}} \tag{A-2}$$

$$T_r = L_r / \left[\left(\frac{\Delta\rho}{\rho}\right)_r Z_r\right]^{\frac{1}{2}} \tag{A-3}$$

where U_r is the velocity ratio in model and prototype, T_r is the time ratio, L_r is the horizontal length ratio, Z_r is the vertical scale ratio, and $(\Delta\rho/\rho)_r$ is the density difference ratio. Eqs. (A-1), (A-2), and (A-3) are written for the two-layered lake depicted in Fig. 6. To obtain Froude number similarity for the free surface, Eq. A-1, as well as interfacial Froude number similarity, Eq. A-2, it is necessary for $(\Delta\rho/\rho)_r$ to be unity. Only gravitational forces have been simulated and frictional forces are normally included by a trial and error process and model verification. Coriolis effects are incorporated by rotation of the model as specified by equality of Rossby number in model and prototype. This gives $\omega_r = \left[\left(\frac{\Delta\rho}{\rho}\right)_r Z_r\right]^{\frac{1}{2}}/L_r$, where ω_r is the ratio of angular velocity in model and prototype. The above criteria apply only to the horizontal

mean motion in the two layers and do not account for interfacial friction.

Considering the interfacial region in more detail it is seen that equality of the stability parameter, θ, and Richardson number, R_i, in model and prototype would ensure similarity of frictional and gravitational forces. For the situation depicted in Fig. 4, the Richardson number gives the same condition as Eq.(A-2).

Assuming it is desired to scale the Brunt-Vaisala frequency according to the model time ratio, it is seen that the horizontal and vertical scale ratios must be the same (i.e. $L_r = Z_r$). This in turn requires that $(\Delta\rho/\rho)_r$ be equal to one so that the velocity and time ratios be the same in both layers. Thus, Eqs. (A-1) through (A-3) reduce to $U_r = (L_r)^{\frac{1}{2}}$, $T_r = (L_r)^{\frac{1}{2}}$, and $(\Delta\rho/\rho)_r = 1$.

From the stability parameter, $\theta_r = 1$, we have

$$\nu_r = (L_r)^{3/2} \qquad \text{(A-4)}$$

From this it is seen that the criteria for complete dynamic similarity of the interfacial hydrodynamics in a model of a stratified lake present stringent requirements. These requirements are that the model not be distorted and that the viscosity ratio be given by Eq.(A-4). The first requirement is unrealistic for most lakes assuming reasonable model size. The second requirement cannot be satisfied in practice. It can be concluded that strict dynamic similarity cannot be achieved and therefore hydraulic models of stratified lakes must be operated in accordance with Eqs. (A-1) through (A-3). Boundary friction and wind friction at the free surface may be adjusted but there appears to be no way to adjust the interfacial friction. If possible, δ_r should be scaled according to Z_r. This becomes impractical if $(\Delta\rho/\rho)_r$ is not equal to one.

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APPENDIX B

Interfactial Friction

Assuming that the heavier bottom layer is static and the flowing upper layer has negligible vertical accelerations, the pressure in the two layers can be written as (see Fig. B-1).

$$P_1 = \rho_1 g(h_2 + h_1 - z) \tag{B-1}$$

$$P_2 = \rho_1 g(h_1 + h_2 - z) + \Delta\rho g(h_2 - z) \tag{B-2}$$

Thus the piezometric head in the bottom layer can be written as

$$E_2 = \frac{P_2}{\rho_2 g} + z = \frac{\rho_1}{\rho_2} h_1 + h_2 \tag{B-3}$$

and in the top layer as

$$E_1 = \frac{P_1}{\rho_1 g} + z = h_1 + h_2 \tag{B-4}$$

Assuming that there is no motion in the bottom layer,(i.e. $\partial p_2/\partial x = 0$), one obtains the following relationships:

$$\frac{dh}{dx} = - \frac{\Delta\rho}{\rho_1} \frac{dh_2}{dx} \tag{B-5}$$

$$\frac{dh_1}{dx} = - \frac{\rho_2}{\rho_1} \frac{dh_2}{dx} \tag{B-6}$$

$$\frac{dh}{dx} = \frac{\Delta\rho}{\rho_2} \frac{dh_1}{dx} \tag{B-7}$$

Utilizing Eq.(B-6) it is seen that dE_2/dx is indeed zero. For the flowing upper layer the total energy, H_1, assuming two-dimensional flow or flow in a rectangular cross section is

$$H_1 = h_1 + h_2 + \frac{q^2}{2gh_1^2} \tag{B-8}$$

where q is the discharge per unit width.

Taking the derivative of H_1 with respect to distance and equating this to the slope of the energy grade line ($\frac{dH_1}{dx} = - S_f$), there results

$$-S_f = \frac{dh_1}{dx}\left[\frac{\Delta\rho}{\rho_2} - \frac{q^2}{gh_1^3}\right] \qquad \text{(B-9)}$$

For a rectangular cross section the friction slope can be approximated by assuming a balance of forces over small distances similar to the usual non-uniform free surface flow analysis. This gives

$$S_f = \frac{\tau_i + \alpha\tau_w}{\rho_1 g h_1} \qquad \text{(B-10)}$$

where $\alpha = 2h_1/b$, τ_i is the interfacial shear stress, and τ_w is the wall shear stress. Equating (B-9) and (B-10) it is found that

$$\tau_i + \alpha\tau_w = -\rho_1 g h_1 \frac{dh_1}{dx}\left[\frac{\Delta\rho}{\rho_2} - \frac{q^2}{gh_1^3}\right] \qquad \text{(B-11)}$$

Introducing friction factors, f_i and f_w, defined as

$$\frac{\tau_i}{\rho_1} = \frac{f_i}{8}\frac{q^2}{h_1^2} \qquad \text{(B-12)}$$

$$\frac{\tau_w}{\rho_1} = \frac{f_w}{8}\frac{q^2}{h_1^2} \qquad \text{(B-13)}$$

we obtain finally

$$f_i + \alpha f_w = -8\frac{dh_1}{dx}\left[\frac{1}{F^2} - 1\right] \qquad \text{(B-14)}$$

where $F^2 = q^2/g\frac{\Delta\rho}{\rho_2}h_1^3$.

Equation (B-14) can be used to evaluate interfacial friction for steady non-uniform flow if there is knowledge of $h_1(x)$ and F^2. The wall friction can be calculated based on existing knowledge. For developing boundary layer flows and for smooth walls, f_w will be dependent on R_x (i.e., $R_x = \frac{Ux}{\nu}$). For fully developed flow f_w is a function of R_e where $R_e = U4h_r/\nu$.

Based on a boundary layer type analysis of the interfacial region as depicted in Fig. B-2, the following expression for interfacial shear has been derived [Keulegan (1944), Lock (1951), Bata (1959)]:

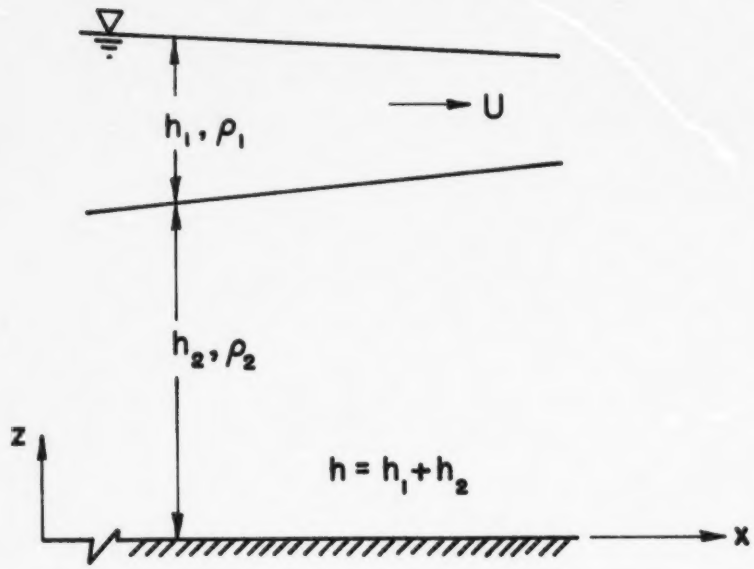

Fig. B-1 Schematic Sketch of Two-Layered System for Steady Nonuniform Flow of Upper Layer

Table B-I

Data from Interfacial Friction Tests

$(\Delta\rho/\rho_2 = .0422)$

Test	q (cm^2/sec)	h_1 (cm)	dh_1/dx	F	f_w^*	f_i	h_r (cm)	R_e
II-B-2	26.7	21.06	.0009	.043	.0305	3.805	5.59	2840
II-B-4	41.5	20.80	.0016	.0679	.0278	2.684	5.57	4460
II-B-5	52.4	20.89	.0032	.0851	.0265	3.427	5.60	5610
II-B-6	67.0	19.99	.0036	.1168	.0251	2.014	5.50	7360
II-B-7	69.4	15.09	.0091	.184	.0236	2.033	5.06	9300
II-B-8	66.4	14.86	.0076	.180	.0236	1.774	5.05	9030
$(\Delta\rho/\rho_2 = .0404)$								
II-A-1	19.1	16.90	.0025	.0713	.0183	3.88	5.25	3880
II-A-2	28.7	16.55	.00375	.0678	.0147	6.438	5.21	3620
II-A-3	40.9	16.60	.00375	.0954	.0123	3.213	5.22	5140
II-A-4	49.8	15.60	.0050	.128	.0109	2.358	5.12	6540
II-A-5	64.3	14.33	.00313	.188	.0092	0.663	4.97	8910

$^*f_w = 0.236/R_x^{1/5}$ [Rouse (1938)]

in series II-B, x = 220 cm

in series II-A, x = 180 cm

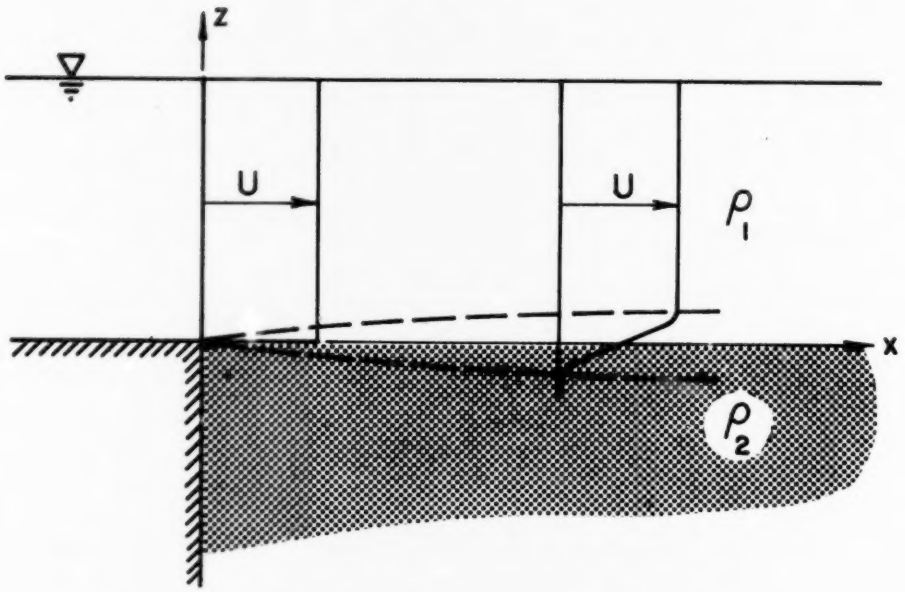

Fig. B-2 Schematic Sketch of Interfacial Boundary Layer Development

$$\frac{\tau_i}{\rho_1 U^2} = \frac{0.2}{R_x^{\frac{1}{2}}} \qquad \text{(B-15)}$$

Utilizing the experimental set-up shown in Fig. 6, a series of tests were conducted with the objective of evaluating f_i based on Eq.(B-14). The experimental data are given in Table B-I.

The calculated values of f_i do not compare favorably with either the earlier test results of Lofquist (1960) or values calculated using Eq.(B-15). The reasons for this discrepancy are not known. Errors in measurement of the experimental data could not account for such large differences.

It was anticipated that the occurrence of breaking interfacial waves would produce changes in τ_i and hence f_i. Although the velocity gradient would be lessened by the rapid mixing, the local eddy viscosity would increase. Thus, it is not clear how much effect the interfacial wave breaking would have on interfacial shear. These preliminary results do not shed much light on this question.

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